Final Exam

December 10, 2024, 3.00 pm

Networks, Crowds and Markets

Universitat Pompeu Fabra
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Name and Surname ....

UNIS .... DNI/ID .... Signature .... Non-done exercise ....

Instructions

- All responses must be justified. Demonstrate your understanding when providing answers.
- The exam duration is 120 minutes.
- You may not leave the room during the first 60 minutes.
- You are permitted to use your own calculator and formula sheet following the provided instructions.
- In the **Test Part**, use the separate answer sheet provided to complete the entire test. Each question will score 0.2, wrong questions will start penalizing from the 6th error onwards. The penalty will be -0.1. Additional instructions for completion will be provided before the exam starts.
- In the **Exercises Part**, choose 2 out of the 3 exercises in the exam. Clearly indicate the exercise you are not answering by crossing it out and listing it on the front page. In case of doubt, only the first 2 exercises will be graded.
- Ensure that you write your name and university code.

Exercises (60 points)

Choose 2 out of the 3 exercises.

1. *(30 points)* **Random Networks**

   Consider a Random Network (Erdös - Rényi Network) $G(N, p)$ with $N = 10^4$ nodes and $p = 2 \cdot 10^{-4}$.

   (a) *(0.5 points)* Compute the expected number of links, $\langle L \rangle$.

   (b) *(0.5 points)* Determine the average degree $\langle k \rangle$.

   (c) *(0.5 points)* What is the proportion of vertices with degree at most 2.

   (d) *(0.5 points)* Compute the expected number of triangles.

   (e) *(1 point)* Compute the clustering coefficient (explain how do you obtain the formula for this computation)

2. *(30 points)* **Scale-Free Networks**

   Consider a Scale-Free Network with $N = 10^4$ nodes, $\gamma = 3.3$, and $k_{\text{min}} = 1$. Calculate:

   a) *(0.5 points)* The average degree $\langle k \rangle$.

   b) *(0.5 points)* The expected number of links.

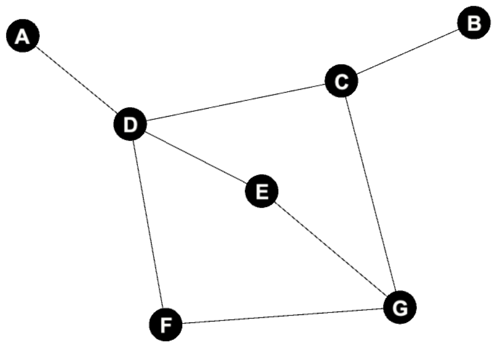
   c) *(0.5 points)* The probability of a node having at most 3 links.

   d) *(0.5 points)* The proportion of nodes with $k \geq 1000$.

   e) *(1 point)* Compute the maximum degree (explain how to obtain the formula for the computation)

3. *(30 points)* **Centrality Measures**

   Given the following network:



   a) *(0.5 points)* Compute the diameter of the network. Where would you add a node H and a link in order to increase the diameter of the network by one unit?

   Do not consider the added node and link for the following parts of the exercise.

   b) *(0.5 points)* Calculate the average degree of the network.

   c) *(0.5 points)* Determine the Closeness Centrality of nodes C and E.

   d) *(0.5 points)* Find the normalized Betweenness Centrality of nodes A and E.

   e) *(1 point)* Add as many links as you need so that the clustering coefficient of node D becomes 1 and prove that the clustering coefficient for a Random Network is constant.

Test (4 points)

Q1. What is the diameter of a graph?

A. The maximum distance between any two nodes in the graph.

B. The minimum distance between any two nodes in the graph.

C. The length of the shortest path in the graph.

D. The average distance between all pairs of nodes in the graph.

Q2. For a Random Network, what is the threshold of $\langle k \rangle$ from which we expect to find a giant component.

A. $\langle k \rangle = 10$

B. $\langle k \rangle = p(N-1)$

C. $\langle k \rangle = \ln(N)$

D. $\langle k \rangle = 1$

Q3. A network has N nodes and average degree $\langle k \rangle$. What does the expression $\frac{\ln N}{\ln \langle k \rangle}$ best represent?

A. The distance between any two nodes in a random network.

B. The average distance between nodes for a random network and some real networks.

C. The diameter for all networks.

D. The average distance for all networks.

Q4. What are some of the conclusions drawn from Milgram's Experiment?

A. There is a lack of short paths in Social Networks.

B. There is no Small World phenomenon in social connections.

C. People effectively find short paths in Social Networks without a global map.

D. Social networks are too vast for effective communication.

Q5. How does the average path length between nodes behave in scale-free networks when $\gamma > 3$?

A. It stays constant regardless of γ.

B. It exponentially increases.

C. It linearly decreases with the number of nodes N.

D. It behaves like the one in random networks.

Q6. In scale–free networks with parameter γ

A. the variance of the degrees of nodes is not well defined for any value of γ (hence the name 'scale–free').

B. the expected degree of a node is not well defined for any value of γ (hence the name 'scale–free').

C. if $2 < \gamma < 3$ there is no concentration of the degree distribution on the expected value (hence the name 'scale–free').

D. the degree distribution can be approximated by a Poisson law for every value of γ.

Q7. What distinguishes a scale-free network from a random network in terms of its degree distribution?

A. Scale-free networks have a uniform degree distribution across nodes.

B. Random networks have a power-law degree distribution.

C. Scale-free networks have many low-degree nodes and a few high-degree hubs.

D. Random networks have a Gaussian degree distribution.

Q8. In the Barabási-Albert model,

A. one can choose the parameter γ of the generated scale–free network by an appropriate choice of the preferential attachment parameter.

B. the degree of a node at time t is proportional to $\sqrt{t}$.

C. the network generated has no power law.

D. the node attached at time t is linked to the existing nodes uniformly at random.

Q9. What is the idea of betweenneess centrality?

A. Vertices are most central if they have a large degree.

B. A vertex is important if it sits on average on a large share of shortest paths between vertices.

C. Vertices that are connected to important edges are more central.

D. A vertex is more important if it is connected to many other vertices by short paths.

Q10. The normalized eigenvector centrality of a node:

A. Approaches 1 the more relevant the neighbours of the node are to the network.

B. Approaches 1 the more well-connected the node is to the rest of the network.

C. Approaches 0 the lower the degree of the node.

D. Approaches 0 the lower its betweenness centrality is.

Q11. What is the difference between Katz centrality and PageRank?

A. Katz centrality has a free parameter α, which in practise is often taken to be $\alpha = 0.85$.

B. PageRank orders the edges by decreasing centrality.

C. In PageRank the centrality derived from a network neighbour is proportional to its centrality divided by its degree.

D. Katz centrality works only for undirected graphs.

Q12. How does the Kernighan–Lin Algorithm work?

A. It computes in each step the optimal cut among a set of partitions obtained from local changes.

B. It iteratively removes edges until the cut is minimised.

C. It implements a random greedy heuristic.

D. It performs an exhaustive search over all graph partitions to determine the minimum cut.

Q13. What does Menger's theorem tell us?

A. A graph has a perfect matching if every vertex has the same degree.

B. If a graph is well-connected, then there are many pairwise internally disjoint paths between any two vertices.

C. If there is a path with k edges between two vertices, then one has to remove at least k vertices to disconnect the two vertices.

D. The minimum cut of a graph is determined by the number of alternating paths.

Q14. In terms of social networks, a *local bridge* is defined as:

A. A tie that strongly connects two friends in a social network.

B. An edge whose removal leads to an increase in the number of components of the network.

C. An edge that maximises the betweeenness centrality.

D. An edge whose endpoints have no neighbours in common.

Q15. In terms of social networks, what does the *The Strength of Weak Ties* theory predict?

A. Weak ties always become strong ties over time.

B. Strong connections between nodes are more likely to weaken over time.

C. In the presence of strong ties, local bridges are formed by weak ties.

D. Strong connections tend to be more reliable than weak connections.

Q16. For a signed graph that satisfies the (general) structural balance property, it follows that:

A. The number of positive edges on cycles is even.

B. There are no triangles exactly with two negative edges.

C. We may partition the vertex set into two parts such that all crossing edges are positive.

D. There may be some negative edges.

Q17. How does the Girvan–Newman Algorithm work?

A. It iteratively removes edges with high centrality.

B. It adds random edges to form parts.

C. It removes vertices belonging to small components.

D. It looks for disconnected vertices.

Q18. When does a graph have a perfect matching?

A. If it is bipartite with boths parts have equal size.

B. If all sets of vertices are constricting.

C. If there is an isolated vertex.

D. If it is regular and bipartite.

Q19. Define constricted sets in the context of the bipartite matching problem.

A. A set of vertices whose neighborhoods coincide.

B. A set of vertices in which every vertex has at least one neighbour.

C. A set of vertices with the same degree.

D. A set of vertices whose neighbourhood is smaller than the set itself.

Q20. How can we use Hall's theorem in auction theory?

A. It allows us to find a stable matching.

B. Given a set of buyer valuations, we can identify market clearing seller prices.

C. We can use it to compute the size of a largest constricting set.

D. Given a set of seller prices, we can identify market clearing buyer valuations.